Approximation and Control of Large-Scale Dynamical Systems

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Abstract

Dynamical systems are a principal tool in the modeling, prediction, and control of physical phenomena with applications ranging from wave propagation and vibration suppression in large structures to timely prediction of storm surges before an advancing hurricane. Direct numerical simulation of associated these models may be the only possibility for accurate prediction or control of complex physical phenomena. However, the ever increasing need for improved accuracy requires the inclusion of ever more detail in the modeling stage, leading inevitably to ever larger-scale, ever more complex dynamical systems.

Simulations in such large-scale settings can be overwhelming and make unmanageably large demands on computational resources, which is the main motivation for *model reduction*. Using the systems-theoretic techniques, model reduction aims to produce a much lower dimensional system whose input/output behavior mimics that of the original as closely as possible. Low dimensionality of the model implies far less storage and far less evaluation time is required. The resulting reduced order model can then be used as an efficient surrogate to the original, to replace it in a larger simulation or to develop a simpler and faster controller suitable for real time applications.

In recent years, interpolatory model reduction methods have emerged as extremely effective strategies for large scale problems. The main idea behind this class of methods is to generate a reduced-model whose transfer function interpolates that of the original one at selected interpolation points along selected interpolation directions. Interpolatory model reduction methods require only solving large sparse linear systems without any dense matrix operations and are capable of producing, at least locally, optimal reduced models. In these lectures, we will not only investigate the fundamentals of interpolatory model reduction but also the most recent developments in this direction. A tentative list of topics are as follows:

- The general projection framework for interpolatory model reduction,
- How to choose the interpolation points and interpolation directions optimally,

- Structure-preserving interpolation and model reduction of systems with generalized coprime factorizations,
- Interpolatory model reduction of Differential Algebraic Equations

The projection-based model reduction requires access to internal dynamics of the original model. However, in some applications, only measurements of system response are available. This motivates a *data-driven* approach to model reduction strategies, whose goal is to produce a reduced model using only accessible simulation inputs and outputs. A tentative list of topic we will study under this category are

- Loewner framework for data-driven interpolatory reduced-order modeling
- How to construct (locally) optimal reduced models via data-driven interpolation

For broad class of problems, the equations representing the underlying state-space dynamics depend on a set of parameters. Such parameters may represent, for example, the variability of material properties, shape, or boundary conditions. If time allows, we will also investigate

- Interpolatory model reduction methods for parametric dynamical systems.