Estimates on weak solutions of semilinear hyperbolic systems

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Abstract. We consider the Cauchy problem for a semilinear hyperbolic system of the type

$$\begin{cases} \partial_t \mathsf{u}(t,\mathbf{x}) + \sum_{k=1}^{d} \mathbf{A}^k(t,\mathbf{x})\partial_k \mathsf{u}(t,\mathbf{x}) = \mathsf{f}(t,\mathbf{x},\mathsf{u}(t,\mathbf{x})) \\ \mathsf{u}(0,\cdot) = \mathsf{v} \end{cases},$$

with each matrix function $\mathbf{A}^k$ being diagonal, bounded and locally Lipschitz in $\mathbf{x}$. Discrete models for the Boltzmann equation furnish examples of such systems. For bounded initial data, and right hand side that is locally Lipschitz in u, local existence and uniqueness results in L^∞ are well known, together with some estimates on weak solutions. However, these estimates are not precise enough for some homogenisation problems.

More precise estimates for weak solutions of the above Cauchy problem will be given, supplemented by estimates on the maximal time of existence for the solution. The local existence and uniqueness in L^p setting $(1 < p < \infty)$ will be addressed as well.

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