Boundary conditions for Friedrichs' systems

Krešimir Burazin & Nenad Antonić

Abstract. Symmetric positive systems (Friedrichs' systems) of first-order linear partial differential equations were introduced by Kurt Otto Friedrichs (1958) in order to treat the equations that change their type, like the equations modelling transonic fluid flow. It consists of a first order system of partial differential equations (of a specific type) and an *admissible* boundary condition enforced by a matrix-valued boundary field. Friedrichs showed that this class of problems encompasses a wide variety of classical and neoclassical initial and boundary value problems for various linear partial differential equations.

Based on the needs to adapt the finite element method to these problems, there has been a renewed interest in the theory of Friedrichs' systems recently, which resulted in their new interpretations in the terms of abstract Hilbert spaces, and a new way of representing the boundary conditions has been introduced. The admissible boundary conditions have been characterised by two intrinsic geometric conditions in the graph space, as well as via boundary operators.

While the new abstract approach has provided a simplified framework, its relation to the classical Friedrichs' well-posedness result is still open.

As a first step, we shall investigate under which assumptions do the classical matrix-valued boundary fields determine the boundary operator. The viability of assumptions will be tested on classical examples.

Krešimir Burazin, Department of Mathematics, University of Osijek, Croatia.
e–mail: kburazin@mathos.hr

Nenad Antonić, Department of Mathematics, University of Zagreb, Croatia.
e–mail: nenad@math.hr