Homogenisation theory for Friedrichs systems

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Abstract. General homogenisation theory was originally developed for the stationary diffusion equation. Considering a sequence of such problems, with common boundary conditions, the homogenisation theory asks the question of what form is the limiting equation? The notions of G-convergence of corresponding operators, and H-convergence (also known as strong G-convergence) of coefficients were introduced. Later, the similar questions were studied for parabolic problems, linearized elasticity problems etc.

As Friedrichs systems can be used to represent various boundary value problems for (partial) differential equations, it is of interest to study homogenisation in such a wide framework, generalizing the known situations. Here we introduce concepts of G and H-convergence for Friedrichs systems, give compactness theorems under some compactness assumptions, and discuss some other interesting topics, such as convergence of adjoint operators, topology of H-convergence and possibility for appearance of nonlocal effects. Finally, we apply this results to the stationary diffusion equation, the heat equation, the linearized elasticity system, and a model example of first order equation leading to memory effects. In the first three cases, the equivalence with the original notion of H-convergence is proved. Here the Quadratic theorem of compensated compactness is used in order to verify our compactness assumptions.

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